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A RELATIONAL EVENT FRAMEWORK FOR SOCIAL ACTION

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Social behavior over short time scales is frequently understood in terms of actions, which can be thought of as discrete events in which one individual emits a behavior directed at one or more other entities in his or her environment (possibly including himself or herself). Here, we introduce a highly flexible framework for modeling actions within social settings, which permits likelihood-based inference for behavioral mechanisms with complex dependence. Examples are given for the parameterization of base activity levels, recency, persistence, preferential attachment, transitive/cyclic interaction, and participation shifts within the relational event framework. Parameter estimation is discussed both for data in which an exact history of events is available, and for data in which only event sequences are known. The utility of the framework is illustrated via an application to dynamic modeling of responder radio communications during the early hours of the World Trade Center disaster.

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1. INTRODUCTION

Human activity over short time scales is frequently understood in terms of *actions*, which can be thought of as discrete events in which one individual emits a behavior directed at one or more other entities in his or her environment (possibly including himself or herself) (Bates and Harvey 1975). For instance, participants in a conversation may direct speech, gestures, or movement toward objects, locations, or other persons in the environment (Goffman 1967). Actions need not be solely, or even primarily, communicative: for instance, a paramedic interacting with an injured patient may direct both conversational and physical actions toward him or her, move objects in the environment (e.g., removing clothing or applying a compress), or travel to another location (e.g., a nearby ambulance) to obtain additional tools. Nor must actions take place entirely within the physical domain; careers are readily modeled as sequences of actions such as job changes and workforce entry/exit (see Abbott [1995] for this and many similar examples), and Heise (1991) provides an empirical example in which the research and publication process is itself represented as a series of institutionally defined actions. Actions and their effects rarely exist in isolation, generally being assumed to have social meaning for the actor and/or outside observers (Goffman 1963a) and often being constrained by physical, institutional, or psychological constraints (Abell 1987; Heise 1989). Processes posited to explain such behavior vary widely, ranging from imitation (Miller, Butts, and Rode 2001) and culturally defined "programs" or "scripts" (Axten and Skvoretz 1980) to learning (Macy 1991) and optimization (Coleman 1990). While many of these perspectives are in substantial disagreement about the mechanisms that underlie action, they arguably have a mutual intersection around a common, "core" notion of action per se as a *directed behavioral event, potentially contingent on past history* (Heise and Durig 1997).

This suggests, from a modeling perspective, that a framework centered on this minimal conception of social action—but with the capacity to incorporate various competing theoretical "overlays"—is likely to prove especially fruitful (a point of view advocated by Bates and Harvey [1975], Heise [1989], and Fararo and Skvoretz [1986], among others). To be useful, such a framework must also permit inference from behavioral data, so as to allow for the estimation of the relative strengths of potential mechanisms as well as for principled selection among competing models. This is a particularly central concern, since prior work in this area has generally resulted in formal systems which are deductively sophisticated (classic examples include Skvoretz and Fararo [1980]; Skvoretz [1984]; Abell [1987]; and Heise [1989]), but which have lacked a correspondingly well-developed inferential theory. Such systems have proved to be conceptually powerful but difficult to deploy outside of very restricted empirical settings. Alternately, models based on assumptions of stationarity (e.g., Markov chains) or purely data analytic approaches (e.g., distance-based sequence comparison) are respectively limited in their ability to capture complex temporal dependence (on the one hand) and to provide a principled basis for parameter estimation and uncertainty assessment (on the other). (See Abbott [1995] and Abbott and Tsay [2000] for reviews in the context of sequence analysis, and Levine [2000] and Wu [2000] for related critique.) Here, we introduce a framework that attempts to preserve many of the assets of the action-oriented approach without neglecting the problem of inference, organizing our treatment around a formal notion of *relational events* (discussed below). The proposed framework allows for the modeling of complex dependence among actions, differential treatment of action by type, nonstationary behavior, and influences due to exogenous covariates. Dependencies among actions may be parameterized so as to allow very general effects of past history on present behavior, including contingent constraints on which behaviors may possibly be enacted. The framework also supports likelihood-based inference, thereby facilitating the empirical evaluation of competing explanations for social action within particular settings.

To illustrate the practical utility of the relational event framework, we also demonstrate its use within a specific context. In particular, we here apply the relational event framework to the problem of modeling communication behavior within emergency settings. Actors operating within such settings face severe environmental constraints, as well as cognitive limitations (e.g., narrowed attention [Hockey 1979]) and an unstable social context (e.g., due to disrupted role performance [Dynes 1970:176]). Nevertheless, both trained and untrained actors will attempt to respond to hazardous conditions, taking action to obtain safety for themselves and others (Quarantelli 1960; Mileti et al. 1975; Abe 1976; Noji 1997). While many factors affecting behavior in crisis settings have been identified through past field studies (e.g., see Drabek [1986] for a review), sorting through them has proven difficult: without a systematic way to combine and compare mechanisms, it is rarely possible to adjudicate among competing explanations (much less to make quantitative statements of relative importance). Using the modeling framework developed here, such comparisons can be performed using data (such as radio transcripts or event logs) that can often be obtained in field settings. Here, we illustrate this potential via the application of the relational event framework to radio communication data collected on the World Trade Center (WTC) disaster by Butts et al. (2007).

The overall structure of the paper is as follows. We begin by describing the general modeling framework, deriving the form of the likelihood function in the case when full temporal information is available regarding actors' behaviors. We then consider the special case in which only sequential information is available (i.e., without detailed information on event timing), and show that the model can still be identified up to a pacing constant. For both cases, the substantive content of a relational event model is determined by a broad family of sufficient statistics expressing the effect of past history and/or exogenous influences on future behavior; we thus follow the above by a discussion of several families of such statistics, including both formal definition and substantive motivation. Parameter estimation and computational issues are then discussed. Finally, we close with an illustrative application of the relational event framework to radio communication in the World Trade Center disaster.

2. RELATIONAL EVENT MODEL

As we have emphasized, generalized modeling of behavior in settings ranging from conversation to crisis situations requires the ability to incorporate a wide assortment of cognitive, behavioral, and social/contextual processes. To that end, we provide here a framework for the dynamic modeling of social action that is capable of incorporating all three factors; in this way, our approach can be seen as following within the broad tradition of agent-based models, which seek to capture social action by direct modeling of microdynamics. (See Macy and Willer [2002] for a recent review of sociological uses of this approach.) Unlike traditional agent-based modeling schemes, our approach draws upon event-history analysis (e.g., Blossfeld and Rohwer [1995]) to formulate models that can be fit directly to data. On the other hand, our approach is also unlike that employed in most familiar statistical contexts, in that we allow for complex structures of historical dependence among observed events. Such "hybrid" model families are of increasing importance within the social sciences, particularly when dealing with phenomena such as social networks (e.g., see Snijders [1996]; Robins et al. [2001]; Wasserman and Robins [2005]; Hunter and Handcock [2006]) or location systems (Butts 2007a) in which system components are nearly always interactive. By building models that are both theoretically informed and inferentially tractable, we hope to obtain important new insights into the processes underlying social dynamics.

The central element of our modeling approach is the *relational event*, or *action*, which is defined as a discrete event generated by a social actor (the "sender") and directed toward one or more targets (the "receivers," who may or may not be actors themselves); without loss of generality we restrict ourselves to the single sender/receiver case. (To treat simultaneous joint action by multiple senders and/or receivers, we simply create one or more "virtual" senders and/or receivers that represent subsets of the original sender/receiver set.) We represent actions by tuples of the form $a = (i, j, k, t)$, where $i \in \mathcal{S}$ represents the *sender* of the action, $j \in \mathcal{R}$ represents the *receiver* of the action, $k \in \mathcal{C}$ represents the *action type*, and $t \in \mathbb{R}$ represents the *time* at which the action is taken. For purposes of the present development, we will assume that each action is associated with a single time point. As noted above, the elements of $\mathcal{S}$ and $\mathcal{R}$ need not correspond to individual agents—collective entities, sets of individuals, or even inanimate objects may constitute potential senders or receivers of action, depending on the system being modeled. For convenience, we also define functions s, r, c, and τ, which return (for any given action) the sender, receiver, action type, and time (respectively).

Given a time-ordered set of actions $a_1, a_2, \ldots$, let the set $A_t = \{a_i : \tau(a_i) \leq t\}$ consist of all actions taken on or before time t. For convenience, we also define a *null action*, a_0, such that $\tau(a_0) = 0$, and (without loss of generality) take $\tau(a_i) \geq 0 \, \forall \, a_i \in A_t$. The null action serves as a placeholder for the onset of events in the process under study; while we assume that $a_0 \in A_t$, we will condition our analyses on the realization of a_0 (i.e., we will treat the onset of observation as exogenously determined). While a_0 is fixed, the other events in A_t are stochastic, and our interest will be in modeling them.

For the family of models proposed here, we assume that actions occur via an inhomogeneous, locally Poisson-like process such that actions arise independently conditional on the realized history of previous actions (along, possibly, with covariates). Of course, not all actions arise at the same rates, and the rate structure itself may evolve endogenously based on the events that are realized; thus, it should be emphasized that the assumption of conditional independence of events does not imply marginal independence, nor temporal independence. Rather, we more modestly assume that past history creates the *context* for present action, forming differential propensities for relational events to occur. Once an event occurs, this alters the context of action, and the process begins anew. Put another way, the conditional independence assumption can be understood as presuming that current events are not subject to (1) uncontrolled third-variable effects, (2) influences from the realization of future events (i.e., reverse causality), and (3) influences from the nonoccurrence of other events since the last realized event. Where third-variable effects are expected, they may be controlled either by employing them as covariates, or (in some cases) by appropriate use of fixed or random effects. Thus, the main substantive constraints employed here are those of no reverse causality and no influence from current nonevents. True reverse causality would be a highly exotic assumption (to say the least), but apparent reverse causality can result from phenomena such as errors in recorded event timing or certain types of third-variable effects (e.g., unobserved information diffusion regarding an action that has been determined but has not yet been taken). Where such processes are at work, care must be taken to ensure that the data set includes all causally relevant information. Influence from contemporaneous nonoccurrence can similarly appear as an artifact of missing data or third-variable effects, but can also occur due to underlying social processes. For instance, in strategic settings, the observation that an opponent has not exploited a potential weakness may change one's model of the opponent's knowledge, thereby altering one's own propensity to engage in a particular action. Thus, systems in which actors are strategically oriented, able to engage in significant forward-looking behavior, and have substantial time for observation and reflection may not be well-suited for modeling via this framework. (Since current evidence suggests that forward-looking behavior is usually limited even in highly incentivized strategic settings (e.g., see Camerer [2003]), this may be less of a problem than might be feared.) By contrast, systems in which actors must respond quickly, are unable to engage in forward-looking behavior, and/or lack strategic orientation are less likely to violate the independence assumption. Alternately, systems for which behavior is strongly influenced by past events may be well-approximated by the conditional independence assumption even where some contemporaneous nonoccurrence effects exist: so long as the latter effects are small compared with the effects of the former (in the sense of relative changes to the event hazard), the system will approximate the conditionally independent case.

In addition to determining the relative rates at which current events transpire, the history of past events can also affect which actions are possible—for instance, certain actions may remove a potential target from the receiver set, or enable new types of actions to be taken. To allow for such constraints to be satisfied, we define the support set $\mathbb{A}(A_t) \subseteq \mathcal{S} \times \mathcal{R} \times \mathcal{C}$ to be the set of all sender/receiver/type combinations that are possible at time t, given the realized history A_t. The specific form of this dependence is taken to be exogenous and known, but is otherwise largely unrestricted; the two key assumptions utilized here are that the number of possible actions is finite for all realized histories, and that the set of possible actions is fixed between events (i.e., context shifts can change the set of possible actions, but such changes are fixed until the next event occurs). Both of these assumptions can be relaxed, but we do not pursue such possibilities here. In addition to accounting for any dynamic constraints on possible actions taken, $\mathbb{A}$ may also incorporate invariant limitations on the set of realizable events. For example, an action type such as "repair" may be directed by a human actor toward a vehicle, but it is rarely feasible for a vehicle to "repair" a human actor.

The above preliminaries in hand, we are now ready to specify the likelihood function for a realized relational event history. As is often the case in building event-based models, we begin by specifying the process in terms of its *hazard* and *survival* functions (e.g., see Blossfeld and Rohwer [1995]). For an arbitrary random variable X with probability density function f and cumulative distribution function F, the survival function S is defined by $S(x) = 1 - F(x)$ (i.e., the probability that $X > x$). The hazard function, h, is similarly defined as $h(x) = f(x)/S(x)$, which can be interpreted as the conditional likelihood that $X = x$ given that $X \geq x$ (in our case, the likelihood of an event occurring at a particular time, given that it has not already occurred before that time). We have already stated that we assume the generative process for A_t is such that the realization of each non-null event can be treated as independent given the events that transpired previously. We may thus note that the conditional likelihood for the ith event (denoted a_i) transpiring at time $\tau(a_i)$ will be equal to the hazard for a_i at $\tau(a_i)$ (i.e., the likelihood that a_i transpires at $\tau(a_i)$, given that it has not already transpired) multiplied by the survival functions for all potential events over the time interval from the previous event (a_{i-1}) to the current event a_i (i.e., the joint likelihood that none of the possible events transpired from $\tau(a_{i-1})$ to $\tau(a_i)$). The joint likelihood for the entire event history at some given time t will then be the product of each successive conditional likelihood, along with a final factor accounting for the gap between the time of the last observed event and the end of the observation period. For brevity of notation, let us denote the likelihood of A_t by $p(A_t)$, allowing M to refer to the number of non-null events in A_t and X_a to refer to a covariate set associated with event a. We then posit that the likelihood for the relational event history must take the following form:

$$p(A_t) = \left[\prod_{i=1}^{M}\left[\begin{array}{l} h\big(\tau(a_i) \mid s(a_i), r(a_i), c(a_i), X_{a_i}, A_{\tau(a_{i-1})}\big) \times \\ \displaystyle\prod_{a' \in \mathbb{A}(A_{\tau(a_i)})} S\big(\tau(a_i) - \tau(a_{i-1}) | s(a'), r(a'), c(a'), X_{a'}, A_{\tau(a_{i-1})}\big) \end{array}\right]\right] \times \left[\prod_{a' \in \mathbb{A}(A_t)} S\big(t - \tau(a_M) \mid s(a'), r(a'), c(a'), X_{a'}, A_t\big)\right]. \quad (1)$$

Intuitively, equation (1) traces the history of A_t, incorporating both the likelihoods of events that did occur (the elements of A_t) and the likelihoods of the associated "nonevents" (actions that could have been taken in each instant but were not). Given this general form, we may then specify particular subfamilies by appropriate selection of h and S. One obvious choice in this regard is to assume that each potential action has a constant hazard of occurrence given a particular prior event history (i.e., a piecewise constant latent hazard model). This amounts to the assumption that the waiting time from one event to the next is conditionally exponentially distributed, and hence we can posit some rate function λ such that $h(t) = \lambda$ and $S(t) = e^{-\lambda(t-t')}$ for an event transpiring at time t following a prior event at time $t' < t$. λ, in turn, may be a function of sender, receiver, action type, and past action history, as well as exogenous covariates. A considerable virtue of this approach is that λ may be flexibly employed to incorporate a wide range of endogenous and exogenous influences; the corresponding challenge is then to identify rate functions that are theoretically appropriate. While many choices of λ are possible, certain properties do suggest themselves as starting points; some of these are described in Section 2.2.

In addition to features of the potential action, covariates, and past history, it seems reasonable in many settings to presume that λ will depend on some vector of unknown parameters, θ. To denote the piecewise constant rate function in this general case, we employ the abbreviated notation $\lambda_{aA_t\theta} = \lambda(s(a), r(a), c(a), X_a, A_t, \theta)$. Under the piecewise constant hazard model, we may now substitute the implied definitions of h and S (given above) into equation (1), thereby obtaining the likelihood

$$p(A_t \mid \theta) = \left[\prod_{i=1}^{M}\left(\lambda_{a_i A_{\tau(a_{i-1})}\theta} \prod_{a' \in \mathbb{A}(A_{\tau(a_i)})} \exp\left(-\lambda_{a' A_{\tau(a_{i-1})}\theta}(\tau(a_i) - \tau(a_{i-1}))\right)\right)\right] \times \left[\prod_{a' \in \mathbb{A}(A_t)} \exp\left(-\lambda_{a' A_t \theta}(t - \tau(a_M))\right)\right]. \quad (2)$$

Where λ incorporates unknown parameters, equation (2) may be used to estimate them (see Section 2.3). This is, however, contingent upon fully observed timing information. For data of the sort frequently available, this assumption is problematic: in many settings, all that can be accurately obtained is the order in which events transpire. Before turning to the question of how λ may be parameterized, then, we must first determine how the action model may be adapted to data for which timing information is more limited.

2.1. *Ordinal Data Likelihood*

Where A_t is fully known, the likelihood of equation (2) provides an adequate basis for subsequent inference (see Section 2.3). In general, however, this is not the case—while we may know the *order* in which events occur, we do not always have access to exact interevent times. This is especially true when working with transcript data (like that employed in Section 3), for which the exact timing associated with speech events may not have been recorded. If we assume that τ is known only up to an order-preserving transformation, what can be said regarding the likelihood of A_t? Plainly, equation (2) cannot be used directly, as the interevent time intervals $(\tau(a_i) - \tau(a_{i-1}))$ are not invariant under the appropriate class of transformations. Indeed, if events in A_t can only be ordered, it follows that the associated likelihood can only be based on which of the various possible events appears next in the τ-induced sequence. For the piecewise constant hazard model, it happens that just such a result can be obtained. We begin by noting that, under the model of equation (2), the waiting time for any given event a_i following some event a_{i-1}, conditional on the nonoccurrence of all other events, is exponentially distributed with parameter $\lambda_{a_i A_{\tau(a_{i-1})}\theta}$. The probability that a_i is the first of the possible events to occur is equivalent to the probability that the waiting time for a_i is the minimum waiting time for all potential events;[1] thus, under the piecewise constant hazard model, the probability that a_i occurs first is equal to the probability that a random variable $W(a_i, A_{\tau(a_{i-1})}, \theta)$ is equal to $\min_{a' \in \mathbb{A}(A_{\tau(a_i)})} W(a', A_{\tau(a_{i-1})}, \theta)$ where $W(a, A_t, \theta) \sim \mathrm{Exp}(\lambda_{a A_t \theta})$. This probability, in turn, is obtained via the following theorem:

Theorem 1. *Let $X_1, \ldots, X_n$ be independent, exponentially distributed random variables with rate parameters $\eta_1, \ldots, \eta_n$. Then,* $\Pr(x_i = \min\{x_1, \ldots, x_n\}) = \eta_i / \sum_{j=1}^{n} \eta_j$.

Proof. Without loss of generality, let $Y = \{X_j : j \neq i\}$ refer to the X variables other than X_i; for clarity of notation, we will relabel the elements of this set as $Y_1, \ldots, Y_{n-1}$ with associated rate parameters $\eta'_1, \ldots, \eta'_{n-1}$. By definition, then, $\Pr(x_i = \min\{x_1, \ldots, x_n\}) = \Pr(X_i < \min\{Y_1, \ldots, Y_{n-1}\})$. From the definition of the exponential density, this gives us

$$\begin{aligned}
&\Pr(X_i < \min\{Y_1, \ldots, Y_{n-1}\}) \\
&\quad = \int_0^\infty \int_{x_i}^\infty \cdots \int_{x_i}^\infty \eta_i e^{-\eta_i x_i} \left(\prod_{j=1}^{n-1} \eta'_j e^{-\eta'_j y_j} \mathrm{d}y_j \right) \mathrm{d}x_i.
\end{aligned}$$

Note that since the Y variables depend only on the value of X_i, we may safely integrate them out, giving us

[1]Since we are assuming a continuous waiting time distribution, we may ignore the probability-zero case in which two events occur at precisely the same instant.

$$= \int_0^\infty \eta_i e^{-\eta_i x_i} e^{-\sum_{j=1}^{n-1} \eta'_j x_i} \mathrm{d}x_i.$$

Integrating over the range of X_i, we have

$$= \frac{-\eta_i}{\eta_i + \sum_{j=1}^{n-1} \eta'_j} e^{-\left(\eta_i + \sum_{j=1}^{n-1} \eta'_j\right) x_i} \Big|_0^\infty,$$

which reduces simply to

$$= \frac{\eta_i}{\eta_i + \sum_{j=1}^{n-1} \eta'_j}.$$

Since, by construction, $\sum_{j=1}^{n-1} \eta'_j = \sum_{j=1}^{n} \eta_j - \eta_i$, we may rewrite this expression as

$$= \frac{\eta_i}{\sum_{j=1}^{n} \eta_j},$$

which completes the proof.

Theorem 1 provides us with a surprisingly simple result: the probability that a particular event a_i will be the next in an event sequence (under the piecewise constant hazard model) is equal to the occurrence rate for a_i, divided by the sum of the rates for all possible events that might occur (including a_i itself). Since successive events are conditionally independent, it follows that the likelihood of A_t under temporally ordinal data is merely a product of multinomial likelihoods. Specifically, we have

$$p(A_t \mid \theta) = \prod_{i=1}^{M} \left[\frac{\lambda_{a_i A_{\tau(a_{i-1})} \theta}}{\sum_{a' \in \mathbb{A}(A_{\tau(a_i)})} \lambda_{a' A_{\tau(a_{i-1})} \theta}} \right]. \tag{3}$$

(Note that, although the above is now a probability mass rather than a probability density, as in equation (2), we continue to refer to the likelihood of A_t generically by $p(A_t)$.)

Ironically, this expression is even simpler than that of equation (2). This simplicity is not without cost, however: in addition to the fact that information is lost in the conversion from ratio to ordinal scaling, the particular form of equation (3) allows λ to be identified only up to a constant factor. In practice, this is not a terribly onerous restriction, since we are generally interested in relative rates rather than absolute pace of interaction (though see Moody et al. [2005] for a defense of the value of pacing information). However, it should be noted that this does affect extrapolative simulation, in that the average rate at which actions occur cannot be determined without additional information. It should thus be emphasized that exact timing information should be used where available, although the relational event model can be usefully applied to data for which only order is known.

2.2. *Specification of the Rate Function*

While equations (2) and (3) provide expressions for the stochastic component of the relational event model, the dynamic evolution of the event system itself is driven primarily by the rate function, λ. As indicated above, we presume that λ may in general depend upon sender, receiver, action type, past event history, and/or exogenous covariates, in addition to unknown parameters. Substantively, such dependence allows us to accomplish a number of modeling goals. First, it is desirable to be able to incorporate sender and receiver effects—that is, differential tendencies for certain persons or objects (or persons/objects with certain properties) to send or receive action. Second, it is important to be able to include individual or dyadic covariates (e.g., personal attributes, similarity, or physical proximity), which may impact the chance of an action from one entity to another. Third, the history of past action should impact future behavior, in accordance with known cognitive and behavioral principles. To capture such phenomena within a flexible, interpretable framework, we parameterize the rate function as

$$\lambda(s(a), r(a), c(a), X_a, A_t, \theta) = \exp\left[\lambda_0 + \theta^T u(s(a), r(a), c(a), X_a, A_t)\right], \tag{4}$$

where (as before) a is a hypothetical event, s, r, and c are the source, receiver, and action type functions (respectively), X is a covariate set, and A_t is the past history associated with some time point, t. We presume that $\theta \in \mathbb{R}^p$ is a parameter vector, which is fixed with respect to the evolution of the event system (although possibly uncertain). The function $u : (\mathcal{S}, \mathcal{R}, \mathcal{C}, X, \mathbb{A}) \mapsto \mathbb{R}^p$ is then a vector of sufficient statistics for the model, and $\lambda_0 \in \mathbb{R}$ is a "pacing constant." In general λ_0 gives the temporal scale of the relational event process, acting as the baseline with respect to which other parameters are defined. Under the ordinal data model, λ_0 is arbitrary (and hence may be taken to be equal to zero without loss of generality); we separate it from the θ effects for this reason.

Intuitively, u indexes the various influences that increase or decrease the relative rates of incidence across events. These effects are weighted by θ, such that each unit change in u_i for a potential event multiplies its conditional relative rate by $\exp(\theta_i)$. In this respect, u is directly analogous to the elements of the objective/gratification functions of the dynamic network models of Snijders (1996, 2005), and to the sufficient statistics invoked in the parameterization of exponential random graph (aka ERG or p^*) (Wasserman and Robins 2005; Snijders et al. 2006; Pattison and Robins 2002) and permutation/assignment (Butts 2007a,b) models. As in the case of models proposed for social networks (and in the conditional dependence tradition launched by Besag [1974, 1975]), appropriate selection of sufficient statistics allows us to specify the manner in which one system element depends on another. Here, this dependence is temporal in nature: the propensity of any given actor to direct action to a particular target will (in general) depend upon which actions he or she (or others) have taken in the past, along with exogenous factors (such as personal attributes). These prior actions may be significant not only in their distribution (e.g., how many actions has actor i taken?) but also in their order or timing (e.g., did actor i greet actor j first, or vice versa?). Save for the constraints that u be finite for all actions, that it depend on past history in the manner indicated above, and that its elements be affinely independent (i.e., we cannot recreate u_i from an affine combination of other u elements), there are few limits on what manner of effects may be explored. Since all effects speak directly to the tendency for a given sender to direct action toward a given receiver, they are in a sense closer to the "actor oriented" philosophy articulated by Snijders (2006) than the "tie oriented" philosophy with which he constrasts his approach. As the present family is more naturally parameterized in terms of behavioral propensities than in terms of stochastically maximized utilities, however, a better term for the modeling philosophy employed here would be "behavior oriented." Depending on the agents (and behaviors) being modeled, estimated effects may or may not refer to hypothetical underlying preferences—following Mayhew (1980), we do not impose this interpretation *ex ante*.

Given the flexibility of the set of potential statistics, it can be hard to know how to begin specifying models for practical use—choice of u clearly determines the range of dynamic behavior to be examined, but there are many statistics that might be contemplated. Indeed, comparable developments in the network field have spawned a burgeoning literature on choice of sufficient statistics (e.g., Wasserman and Pattison [1996]; Pattison and Wasserman [1999]; Robins et al. [1999]; Pattison and Robins [2002]; Handcock [2003]; Snijders et al. [2006]; Butts [2006]), a broad implication of which is the importance of selecting statistics that are theoretically appropriate for the process under study (as opposed to the use of a uniform set of standard statistics for all purposes). With this in mind, we focus here upon an initial set of sufficient statistics with clear substantive utility for a particular problem of interest. This problem (as hinted in Section 1) is the analysis of interpersonal radio communication within the context of emergency situations such as the World Trade Center disaster. Although our discussion is thus anchored in a particular application, the statistics defined here can easily be employed in many other settings. Some of these broader applications will be mentioned in the discussion below.

2.2.1. *Fixed Effects*

It is well-known that the distribution of communicative acts within closed group settings tends to be highly unequal (Bales et al. 1951). While such inequality may be endogenous to the communication process, it may also reflect exogenous properties of social actors to some degree (e.g., status characteristics [Smith-Lovin et al. 1986; Fisek et al. 1991]). Within an emergency setting, there are additional reasons to suspect differences among actors in the base tendency to participate in interpersonal communication. Such differential participation may reflect unobserved heterogeneity in situational awareness, training, or institutional role, as well as differences in local context. For instance, responders whose location places them in imminent danger are unlikely to spend long periods of time engaged in radio communication, relative to those whose locations afford them a greater degree of safety. To capture the impact of such factors when they cannot be measured directly, we propose to include fixed effects for participation in the relational event system. To parameterize such effects for a group of size $N = |\mathcal{S}| = |\mathcal{R}|$ communicants, we add N statistics of the form $u_{\mathrm{FE}_m}(a, A_t, X) = I(m \in \{s(a), r(a)\})$, where I is the standard indicator function. The corresponding θ parameters then represent logged rate multipliers for all events having the corresponding individuals as senders or receivers. Such parameters are akin to *degree effects* in dynamic network models, and fulfill the same role as the expansiveness/popularity parameters of the well-known (static) p_1 model (Holland and Leinhardt 1981).

It should be noted that, for the ordinal model (or the exact timing model with λ_0 included), the likelihood will not identify all N fixed effect parameters. This may be easily resolved by fixing one parameter to 0, in which case the others should be interpreted as providing log rate multipliers relative to the reference actor. Other linear constraints (e.g., requiring that the statistics sum to 0) may also be applied, if desired. We employ the former solution for the analyses employed here, treating the first individual in each group as the reference actor.

2.2.2. *Persistence*

Another basic mechanism that is easily captured through the relational event model is *persistence,* or the tendency of past contacts to become future contacts. In particular, let $d(i, j, A_k)$ represent the accumulated volume of communication from actor i to actor j by time k, and let $d(i, A_k)^+ = \sum_{j=1}^{|\mathcal{R}|} d(i, j, A_k)$ be the total outgoing communication volume of actor i. The persistence statistic is then defined by $u_P(a, A_t, X) = d(s(a), r(a), A_t)/d^+(s(a), A_t)$—i.e., the fraction of the sender's prior outgoing communication volume that has been devoted to the receiver. Where the associated θ parameter is positive, this effect produces a tendency for actors to preferentially direct action to those who have comprised the bulk of their past communication history. Such a phenomenon could emerge empirically from unobserved relational heterogeneity as well as from cognitive processes such as the enhanced availability to memory of frequent communication partners (Romney and Faust 1982; Freeman et al. 1987). More broadly, a positive persistence parameter captures a tendency toward social "inertia," in a manner loosely analogous to the role played by a positive AR(1) term in an autoregressive time series process.

While it is most natural to think of persistence as a positive-sign effect, it is also possible to obtain negative persistence parameters. In this case, the model reflects a process of "partner switching," in which actors become less likely (*ceteris paribus*) to contact those who comprise a larger fraction of their past communication history. This could be induced by differential availability of actors over time, as well as by search processes (such as information seeking behavior) which encourage the accumulation of a diverse array of contacts.

2.2.3. *Preferential Attachment*

In the midst of a turbulent environment, judgments regarding potential communication targets may be highly uncertain. When one cannot be sure who is still able to respond, it is natural to utilize past communicative activity as a predictor of current availability: those who have been involved in past communication are more likely to be present and able to respond than those with no prior communicative activity. The phenomenon in which actors with a greater level of past activity are more likely to be chosen as communication targets is an example of *preferential attachment* (Simon 1955; de Solla Price 1965; Barabási and Albert 1999), and is easily captured via a statistic of the form $u_{\mathrm{PA}}(a, A_t, X) = (d^+(r(a), A_t) + d^-(r(a), A_t))/[(\sum_{j=1}^{|S|} d^+(j, A_t)) + (\sum_{j=1}^{|S|} d^-(j, A_t))]$, defining $d^-(i, A_k) = \sum_{j=1}^{|S|} d(j, i, A_k)$ to be the total number of actions received by actor i. Where the associated parameter is positive, actors with more past communication tend to become more attractive targets, creating a positive feedback loop that tends to lead to the creation of high degree actors. By contrast, a negative attachment parameter reflects a tendency to seek out actors who have not had prior involvement with the communication network—for example, due to a novelty seeking process. In this case, the attachment process will tend to suppress the formation of high-degree actors, leading (*ceteris paribus*) to a “flatter” indegree distribution.

2.2.4. *Participation Shifts*

A substantial influence on the structure of communication generally (and radio communication in particular) is the adherence of communicants to *conversational norms* governing communicative acts (Goffman 1967; Sacks et al. 1974; Wilson et al. 1984; Schegloff 1992). These norms include restrictions on the number of recognized speakers (Schegloff 2000) and expectations of reciprocity in turn-taking (Wilson et al. 1984), as well as other constraints on attention and involvement that allow the interaction to be maintained over time (Goffman 1963b). In the context of radio communication using handheld devices, these norms are further strengthened by the intrinsic limitations of the communication technology (e.g., the impossibility of mutually intelligible overlapping talk) as well as organizationally imposed standard operating procedures (Auf der Heide 1989). Such procedures typically require that conversations be structured in terms of a sequence of call/response actions (i.e., turns) including an initial contact query, comprehension checks (and verification thereof), and formal termination of the interaction (sign-off). While formal procedure is not always followed to the letter (especially in an emergency), trained users of a well-monitored channel typically show a communication pattern that is substantially more tightly organized than everyday speech.

Given this, modeling of radio communication in disasters (or conversation more generally) requires a basic means of capturing the effect of conversational norms on communicative action. In important recent work, Gibson (2003, 2005) has proposed quantifying the local temporal dynamics of conversation by counting events known as *participation shifts* (or "P-shifts"). Following Goffman (1981), Gibson (2003) partitions the participants in a conversation into the roles of speaker, target, and unaddressed recipient (i.e., third party). As the conversation unfolds, occupancy of these roles shifts; such shifts are governed by the norms of conversational interaction, and they are the basis for Gibson's analysis. Enumerating the possible shifts in dyadic communication (and allowing for the possibility of a special class of receivers who cannot themselves become senders), Gibson defines 13 distinct P-shifts that can occur during conversational interaction. He denotes these using a two-pair notation, in which the quartad $ij - kl$ denotes the shift from an act in which i sends to j, to one in which k sends an act to l. While Gibson relies on a combination of global incidence counts and permutation tests to examine P-shifts in his data, we can here integrate P-shifts into our more general framework via appropriate choice of sufficient statistics. For this purpose, we must temporarily turn from Gibson's compact notational scheme to a more elaborate one which explicitly articulates the shift definitions in terms of the basic elements of the relational event model; in the text that follows, we will use Gibson's notation to refer to the specified shifts, and the event notation to formally define them. To begin, let a' denote the most recent (i.e., maximum τ) event in A_t, let $\vee$ denote the logical OR operator, let $\wedge$ denote logical AND, and let $\mathcal{R}' = \mathcal{R} \cap \mathcal{S}$ be the set of potential targets who may themselves be senders. Although every action must have *some* target, not every target need be an individual who can send actions of the same type; for instance, it may be useful to represent a radio broadcast to "anyone listening" by a "generic" target that belongs to $\mathcal{R}$ but not to $\mathcal{R}'$. A similar approach may be taken for actions directed at inanimate targets (e.g., speaking into a tape recorder). Where no targets are of this latter kind, then certain P-shifts (described below) cannot occur. Using these basic notions, we define a set of "matching" events, based on whether the sender or receiver of a matches the sender or receiver of a'. Enumerating over all four sender/receiver combinations, these are defined formally as $SS \equiv s(a') = s(a)$, $RR \equiv r(a') = r(a)$, $SR \equiv s(a') = r(a)$, and $RS \equiv r(a') = s(a)$ with respective negations $\overline{SS}$, $\overline{RR}$, $\overline{SR}$, and $\overline{RS}$. In order to establish boundary conditions, we must also define the "targeting" events $T \equiv r(a) \in \mathcal{R}'$ and $T' \equiv r(a') \in \mathcal{R}'$ (negations $\overline{T}$ and $\overline{T'}$) and the compound event $PS \equiv \overline{SS} \vee \overline{RR}$ (i.e., the event that a P-shift occurs). The three "turn receiving" P-shifts identified by Gibson (2003)—AB-BA, AB-B0, and AB-BY in his notation—can then be defined via the statistics

$$u_{\mathrm{AB-BA}}(a, A_t, X) = I(PS \wedge T' \wedge T \wedge SR \wedge RS), \tag{5}$$

$$u_{\mathrm{AB-B0}}(a, A_t, X) = I(PS \wedge T' \wedge \overline{T} \wedge RS), \quad \text{and} \tag{6}$$

$$u_{\mathrm{AB-BY}}(a, A_t, X) = I(PS \wedge T' \wedge T \wedge \overline{SR} \wedge RS) \tag{7}$$

respectively. All three of these P-shifts have in common the conditions that (1) the receiver of the initial event is a potential sender, and (2) the receiver of the initial event is also the sender of the subsequent event. As indicators, their respective statistics take a value of 1 when their associated conditions are met, otherwise taking a value of 0; thus, for instance, $u_{\mathrm{AB-BA}} = 1$ when a is a reciprocating event for a', and 0 otherwise. Similarly, $u_{\mathrm{AB-B0}}$ indicates a shift in which some actor A directs an event toward another actor, B, who in turn directs the next action toward a nonsending target. Finally, where the target of the second action is some actor Y other than A who is a potential sender, a shift of form AB-BY is said to occur. This is captured by the associated statistic $u_{\mathrm{AB-BY}}$, which like the others is a dichotomous indicator for its respective P-shift.

The three "turn claiming" P-shifts (A0-X0, A0-XA, A0-XY) all involve scenarios in which the recipient of the first action is not a potential sender, and have the respective statistics

$$u_{\mathrm{A0-X0}}(a, A_t, X) = I(PS \wedge \overline{T'} \wedge \overline{T}) \tag{8}$$

for the case in which the recipient of the second action is a nonsender,

$$u_{\mathrm{A0-XA}}(a, A_t, X) = I(PS \wedge \overline{T'} \wedge T \wedge SR) \tag{9}$$

for the case in which the second action is directed to the initial sender, and

$$u_{\mathrm{A0-XY}}(a, A_t, X) = I(PS \wedge \overline{T'} \wedge T \wedge \overline{SS} \wedge \overline{SR}) \tag{10}$$

for the case in which the second action is between two entirely different actors (both being potential senders).

Gibson also notes four "turn-usurping" P-shifts, which involve the interruption of a call/response pattern by a new speaker. These have statistics

$$u_{\mathrm{AB-X0}}(a, A_t, X) = I(PS \wedge T' \wedge \overline{T} \wedge \overline{SS} \wedge \overline{RS}), \tag{11}$$

$$u_{\mathrm{AB-XA}}(a, A_t, X) = I(PS \wedge T' \wedge T \wedge SR \wedge \overline{RS}), \tag{12}$$

$$u_{\mathrm{AB-XB}}(a, A_t, X) = I(PS \wedge T' \wedge T \wedge \overline{SS} \wedge RR), \quad \text{and} \tag{13}$$

$$u_{\mathrm{AB-XY}}(a, A_t, X) = I(PS \wedge T' \wedge T \wedge \overline{SS} \wedge \overline{SR} \wedge \overline{RS} \wedge \overline{RR}), \tag{14}$$

with the four shifts being distinguished by whether the target of the second action is a nonsender (AB-X0), the originating sender (AB-XA), the original recipient (AB-XB), or a third potential sender (AB-XY).

Finally, we have the three "turn continuing" shifts (A0-AY, AB-A0, and AB-AY), in which the sender is preserved in each event. The first and second such shifts involve nonsenders as targets, either in the first or second event; the third involves a potential nonsending target. Formally, the statistics for these shifts are defined by the indicators

$$u_{\mathrm{A0-AY}}(a, A_t, X) = I(PS \wedge \overline{T'} \wedge T \wedge SS), \tag{15}$$

$$u_{\mathrm{AB-A0}}(a, A_t, X) = I(PS \wedge T' \wedge \overline{T} \wedge SS), \quad \text{and} \tag{16}$$

$$u_{\mathrm{AB-AY}}(a, A_t, X) = I(PS \wedge T' \wedge T \wedge SS \wedge \overline{RR}), \tag{17}$$

which complete the set of possible P-shifts. (It should be noted that we could also create statistics for the "nonshift" action pairs AB-AB and A0-A0, although we will not do so here.)

Introducing each of these statistics into a relational event model parameterizes the tendency for the system to encourage or discourage the corresponding P-shift. For instance, a large positive coefficient associated with $u_{\mathrm{AB-BA}}$ indicates a strong tendency toward local reciprocity in communication; a negative coefficient, by contrast, would signal a tendency by actors to avoid immediately responding to contacts (as might be expected with actions such as dominance threats (Chase 1980; Chase et al. 1998)). While it is in principle possible to use all statistics simultaneously, the fact that they are nearly affinely dependent may cause problems for parameter estimation. In such cases, it may be wise to remove one or more statistics, in effect adding them to the implied reference group otherwise occupied by "no shift" actions. Likewise, the seven statistics involving nonsending targets (i.e., actions containing $\overline{T}$ or $\overline{T'}$) are applicable only if such actions are permitted within the data set. For strictly dyadic interactions with $\mathcal{R} = \mathcal{S}$ no such events are possible, and hence the relevant P-shift statistics are those corresponding to the six remaining forms.

2.2.5. *Recency*

While turn-taking (as defined above) is a purely local phenomenon, there are also cases in which recency of contact would be expected to have a more general impact on future communication. For instance, there is reason to expect more recent sources to be salient targets for outgoing communication, both due to mnemonic and contextual factors (e.g., repeated coordination demands stemming from an ongoing task). Where multiple conversing subgroups are required to share the same channel (real or virtual), we may also expect occasional interruptions of one conversation by another. In such cases, the interrupted subgroup may be forced to wait until the interrupting subgroup ceases conversing, returning afterward to their ongoing call/response pattern. Such a process can be modeled within the present framework by a statistic such as $u_{\mathrm{R}}(a, A_t, X) = \rho(s(a), r(a), A_t)^{-1}$, where $\rho(i, j, A_t)$ is j's recency rank among i's in-neighborhood. Thus, if j is the last person to have called i, then $\rho(i, j, A_t)^{-1} = 1$. This falls to $1/2$ if j is the second most recent person to call i, $1/3$ if j is the third most recent person, etc. (To ensure that the behavior of ρ is well-defined, actors who do not belong to i's in-neighborhood are considered to have rank ∞.) Note that the use of recency rank relative to the potential sender (as opposed to recency rank over all communications, for example) is based on the "actor oriented" assumption (in the sense of Snijders [2006]) that the salient mnemonic context for a sending actor is his or her prior contacts (rather than all radio traffic); this should be contrasted with the turn-taking recency of the AB-BA P-shift, which is based on the context of the *conversation* rather than the context of the sender's past history. Likewise, the use of ranks rather than an alternative such as number of intervening events is based on the assumption that mnemonic sorting is performed by "chunking" (Miller 1956) incoming communications by sender (making search difficulty scale with the number of intervening senders rather than the number of intervening events). Although this is employed here as a plausible and illustrative parameterization, a more extensive analysis could compare the performance of multiple parameterizations based on various cognitive processes. The ability to perform such comparisons is a useful feature of the proposed approach.

Where the parameter associated with the recency statistic is positive, actors exhibit a tendency to preferentially call those who have most recently contacted them. By turns, a negative parameter value would indicate a tendency to avoid calling those with more recent incoming communications. Such an effect seems unlikely to emerge within a context such as radio communications, but it might be observed for other types of relational events (again, dominance contests being an obvious example).

2.2.6. *Triadic Effects*

The last category of effects considered here consists of those arising from triadic forms. In contrast with those properties already considered (which are, at best, dyadic), triadic effects engender dependencies that are far less local in nature (Frank and Strauss 1986; Strauss 1986). The most famous of these effects is that related to transitive closure (Holland and Leinhardt 1971), which may be understood here as the tendency for the existence of one or more i, j two-paths to enhance or inhibit direct communication from i to j. The impact of the same two-paths on the corresponding j, i communications is naturally understood as a *cyclicity* effect, and it may be motivated by the notion that the target of a brokered communication may be likely to bypass the broker when replying (thus forming a direct connection and creating a cycle). Each of these effects may be parameterized via statistics of the form $u_{\mathrm{OTP}}(a, A_t, X) = \sum_{h=1}^{|\mathcal{R}|} \min\{d(s(a), h, A_t), d(h, r(a), A_t)\}$ (counting outgoing two-paths relative to $s(a)$) and $u_{\mathrm{ITP}}(a, A_t, X) = \sum_{h=1}^{|\mathcal{R}|} \min\{d(r(a), h, A_t), d(h, s(a), A_t)\}$ (counting the corresponding incoming two-paths). The parameters associated with u_{ITP} and u_{OTP} then simply indicate the strength of the tendency to form cycles/transitive closure, or to inhibit the same, depending on the sign of the parameter value.

In addition to the "classic" two-path effects, it is also useful to consider the potential impact of shared partners on direct interaction (Snijders et al. 2006). For instance, two actors who both have contacted the same third parties may be more or less likely to contact one another directly; this is referred to an *outbound shared partner* effect. Similarly, we can imagine an effect due to having been contacted by the same third party, which would constitute an *inbound shared partner* effect. These effects are, respectively, indexed by the statistics $u_{\mathrm{OSP}}(a, A_t, X) = \sum_{h=1}^{|\mathcal{R}|} \min\{d(s(a), h, A_t), d(r(a), h, A_t\}$ and $u_{\mathrm{ISP}}(a, A_t, X) = \sum_{h=1}^{|\mathcal{R}|} \min\{d(h, s(a), A_t), d(h, r(a), A_t\}$. As with the two-path effects, the sign and magnitude of the parameters associated with these statistics indicate the extent to which such configurations are encouraged or inhibited via the dynamic process.

While the above are basic variants, they obviously do not exhaust the possibilities for triadic statistics. For instance, a natural extension of the P-shift statistics introduced above would be to consider second-order shifts—that is, sextads of the form $ij - kl - mn$, with each pair corresponding to the sender/receiver of an event within a three-event sequence. These second-order shifts would then provide a "local" family of triadic effects, much as the first-order shifts allow for the parameterization of local reciprocity. The ability to extend and empirically evaluate schemes such as Gibson's P-shifts using the relational event framework is one illustration of the utility of the general approach.

2.3. *Parameter Estimation*

Given a choice of sufficient statistics, either equation (2)—in the exact case—or equation (3)—in the ordinal case—defines the likelihood of A_t under the relational event model. Since both expressions are readily computable, there is in principle no difficulty in carrying out likelihood-based inference for θ given A_t. (Indeed, the relative ease of likelihood computation for this model family dramatically reduces the difficulties frequently encountered with dynamic tie-based models such as those of Snijders (2005), or static ERGs (e.g., see Handcock [2003] for a description of some of these issues).) The most obvious tactic to employ in this regard is maximum likelihood estimation—that is, identifying $\hat{\theta}$ such that

$$\hat{\theta} = \arg\max_{\theta} p(A_t \mid \theta) \tag{18}$$

using a variant of Newton-Rapheson, simulated annealing, or other heuristic optimization methods (see Acton [1990] for a number of approaches). Once $\hat{\theta}$ has been calculated, the inverse information matrix at the MLE can be employed to obtain approximate standard errors in the usual fashion. Alternately, fully Bayesian estimation of θ can be performed by positing a prior distribution on θ and maximizing and/or simulating draws from $p(\theta \mid A_t) \propto p(A_t \mid \theta)p(\theta)$. Though we do not treat the issue in detail here, simulation of posterior draws for the relational event model is fairly straightforward using a Metropolis algorithm; see Gelman et al. (1995) for an overview of this approach.

One computational challenge that does emerge is the need to calculate the product of survival functions (or sum of rates, in the ordinal case) across all $|\mathbb{A}(A_t)|$ possible events at each iteration. Since this quantity generally scales with $|\mathcal{S}||\mathcal{R}||\mathcal{C}|$, the number of elements involved can quickly get out of hand when the individual set sizes become large; even given a single action type, the complexity of this calculation will generally grow with the square of the number of actors. This problem is a (thinly disguised) version of the normalizing factor computation that makes estimation for ERGs difficult (see Wasserman and Robins [2005]). Although much less severe in our case than the ERG equivalent, the computational cost of evaluating the normalizing factor can still become prohibitive for large problems. In this case, it is often feasible to replace the relevant quantity with a Monte Carlo estimate, based on explicit calculation of a limited number of events. For instance, let $a''_1, \ldots, a''_m$ be drawn uniformly from $\mathbb{A}(A_t)$. We may then approximate the normalizing factor of equation (3) by the Monte Carlo quadrature

$$\sum_{a'} \lambda_{a' A_t X_{a'} \theta} \approx \frac{|\mathbb{A}(A_t)|}{m} \sum_{j=1}^{m} \lambda_{a'' A_t X_{a''} \theta} \tag{19}$$

(e.g., see Kalos and Whitlock [1986]). Depending on the sufficient statistics on which λ depends, stratification of sender, receiver, and/or action type may be required to ensure convergence of the estimated likelihood. In particular, we should beware of any scheme that results in a failure to cover each sender, receiver, and action type (particularly where fixed effects are employed). Stratification can also reduce the variance of the estimator, which can be estimated by the appropriately scaled variance of the sampled rates. As a rule of thumb, the standard deviation of the estimated normalizing factor should be small compared with both $\lambda_{a_i A_{\tau_{a_{i-1}}} X_{a_i} \theta}$ and with the estimated normalizing factor itself. Where this condition is met for all i, the estimated likelihood will closely approximate the exact likelihood, and the resulting estimators should be well-behaved. Otherwise, it may be necessary to increase m and/or employ additional stratification so as to increase the precision of the estimated normalizing factor. Where this method also fails, importance sampling using the Geyer and Thompson (1992) framework (now frequently used in the network modeling literature) provides yet another alternative. While this approach can be relatively labor-intensive to implement (see Handcock et al. [2003] for an example), the added difficulty may be justified when working on very large data sets.

3. SAMPLE APPLICATION: COMMUNICATION IN THE WORLD TRADE CENTER DISASTER

To illustrate the use of the relational event framework, we here apply the ordinal data model to a subset of radio communication data from responders to the World Trade Center disaster obtained and coded by Butts et al. (2007). The World Trade Center disaster occurred on September 11, 2001, when two hijacked airliners were flown into the North and South Towers of the WTC complex. The resulting fires (aggravated by structural damage from the initial collisions) resulted in the collapse of both buildings (as well as WTC 7), killing the remaining occupants. In the period following the initial plane impacts, a substantial response was mounted both by workers in the towers and by personnel at other affected sites (including Newark Airport, the Port Authority Trans-Hudson system, and Lincoln Tunnel). The data employed here (described in detail below) is derived from radio conversations within groups of responders at the WTC and other related sites; the portion analyzed consists specifically of the sequences of communicative actions (i.e., transmissions) taken by responders within each of six groups during the event.

In prior work, Butts and Petrescu-Prahova (2005), Petrescu-Prahova and Butts (2005), and Butts et al. (2007) analyzed the time-aggregated communication structure using conventional network-analytic methods. An important finding from these analyses was the dominance of the aggregate communication networks by a small number of highly central, "hub" nodes whose prominence could not be accounted for simply by organizational role. Such positions are of particular interest for understanding the emergence of coordination within groups during emergency situations, and predicting their incidence is important for problems such as communication system design (see Butts et al. [2007] for a discussion). On the basis of their aggregate analyses (together with arguments from the prior literatures on networks and disasters), these authors suggest several possible mechanisms that could potentially account for the emergence of hub positions. First, there is the obvious possibility of unobserved heterogeneity within the responder population, leading directly to differences in communication frequency. Such heterogeneity could consist not only of differences in individual characteristics, but also differences in individual context within the event—since not all persons would be expected to have equal opportunity to communicate (e.g., depending on the safety of their local environments), it would not be surprising to observe some differences in overall communication rates. An alternative to heterogeneity would be preferential attachment based on airtime: responders with more prior communication may be expected to become more salient targets for other responders, leading to a feedback loop that amplifies small (possibly random) initial differences in communication rates. Yet another possibility is that hubs emerge from local biases in communication, such as a tendency toward persistence in partner selection, reciprocity, triadic effects, or adherence to conversational norms. As with preferential attachment, such local mechanisms might operate alone or in conjunction with individual-level heterogeneity to promote hub formation.

While conventional, cross-sectional analysis of aggregate communication data is effective in identifying candidate mechanisms, it is not well-suited to discriminating among competing (or complementary) dynamic effects. The relational event framework, however, allows us to directly estimate the contribution of each putative mechanism to the entire communication sequence and to select among competing models where appropriate. The analysis presented here is intended to illustrate these advantages by investigating the conversational dynamics of WTC radio communications.

3.1. *Data*

The data employed here are derived from several documents belonging to a larger collection released by the Port Authority of New York and New Jersey and analyzed by Butts et al. (2007). Specifically, the raw materials for this study consist of radio communication transcripts from six groups of responders to the WTC disaster. Each transcript documents all voice communications associated with a single channel; each channel was used exclusively by a single group of responders.[2] The six transcripts employed here encode (in increasing order of length) the Port Authority Trans-Hudson channel 27 (PATH Radio), Newark airport maintenance (Newark Maint), Newark airport police (Newark Police), New Jersey State Police Emergency Network channel 2 (NJSPEN 2), Newark airport command post/dispatch (Newark CPD), and World Trade Center police (WTC Police) channels. Lengths range from 70 to 481 eligible transmissions (see below), with the number of named communicants ranging from 24 to 46. The time period covered by each transcript begins with the impact of the first plane into the North Tower at 8:46 AM, and ends at three hours and thirty-three minutes or (where relevant) until the collapse of the structures containing the communicants (roughly 1 hour and 15 minutes).

3.1.1. *Coding*

Each radio transcript contains a list of transmissions exchanged among responders, presented in chronological order. Some sender information is provided by the transcriber; depending on the specific transcript, this includes some or all of name, rank, gender, and organization. This information, together with transcript content (including communicants' use of names and callsigns, sequence information, and conversational

[2]Examination of transcript content, as well as other supporting materials (including the 9/11 Commission report [National Commission on Terrorist Attacks Upon the United States 2004]) strongly suggests that the groups studied here lacked access to other radio channels. Thus, we treat the two as effectively equivalent for purposes of this study.

cues), was used by Butts et al. (2007) to assign a unique identifier to the sender and named target(s) of each transmission. Where one-to-many communications were encountered, each was coded as a series of dyadic transmissions from the sender to each of the named recipients (in the order named). Transmissions with no clear target(s), and/or targets that were identified only as a group (e.g., "anyone," "all units") are outside the scope of person-to-person communications considered here and were removed from the data set. The resulting lists of ordered transmissions (one per transcript) comprised the event sets (A_t) employed in subsequent analyses. The sets of potential senders and receivers for each transcript ($\mathcal{S}, \mathcal{R}$) were taken to be the union of named communicants from the transcript in question, with τ corresponding to the order of appearance for each transmission event. Finally, because we analyze only radio communications, we consider all relational events to be of the same type (i.e., $|\mathcal{C}| = 1$).

In addition to the relational event data itself, we consider the formal status of individual responders as an illustrative covariate. Specifically, Butts et al. (2007) attempted to identify individuals within each organization whose formal roles entailed coordinative responsibilities. As this was not available from archival sources, such status was inferred from the content of the available transcripts. Butts et al. coded communicants as occupying institutionalized coordinator roles if their transcriber-assigned labels or within-transcript terms of address contained one of the following words: "command," "desk," "operator," "dispatch(er)," "manager," "control," and "base." Within the Newark Airport transcripts, the content of the communications suggested that actors were referring to a centralized Newark Airport command desk as simply "Newark Airport," so this individual was also assigned to institutionalized status.

3.2. *Model Parameterization*

To translate our substantive intuition into the terms of the relational event framework, we identify a series of statistics (and associated parameters) that formally capture the potential dynamics of the system. Section 2.2 discussed a number of such statistics in detail, and we do not recapitulate that discussion here. We do, however, identify the statistics that are used for the analyses at hand, along with the number of parameters in each independent set.

3.2.1. *Individual-Level Heterogeneity*

As noted above, unobserved differences in context, training, or organizational role may render some actors more likely than others to communicate during the event. To capture this, we introduce fixed effect parameters for each actor, with statistics defined per Section 2.2.1. These statistics are entered as a block of dimension $N-1$, where N is the number of actors; for purposes of model identification, the first parameter is set to 0. In subsequent discussion, this block of effects will be referred to by the initials "FE."

3.2.2. *Preferential Attachment*

In a chaotic and uncertain environment, responders overheard engaging in radio communication may become attractive targets for other responders seeking someone with whom to communicate. As described in Section 2.2.3, this may lead to a phenomenon of preferential attachment (wherein responders become relatively more likely to direct calls to those who have more airtime). Here, we parameterize this as a total communication volume effect, of the form given in Section 2.2.3. This yields a single statistic, whose associated parameter is positive when preferential attachment is present (or negative if a form of "personnel rotation" is taking place). In subsequent discussion, this effect will be referred to by the initials "PA."

3.2.3. *Triadic Effects*

As responders interact, they may become motivated to contact others due to their shared interactions with third parties; alternately, third parties that efficiently route information may become effective substitutes for direct contact. To test for these possibilities, we employ the four triad effect statistics described in Section 2.2.6. These effects (globally identified by the initial "T") are denoted "ITP" for incoming two-paths, "OTP" for outgoing two-paths, "ISP" for incoming shared partners, and "OSP" for outgoing shared partners. Where the parameters associated with these statistics are positive, stronger two-path/shared partner connections promote contact among the endpoints; where negative, stronger connections inhibit interaction.

3.2.4. *Cognitive Effects*

In addition to preferential attachment, it is reasonable to presume that more basic perceptual and mnemonic effects will also be at work shaping responder communications. Two examples that can be motivated in this manner are the persistence statistic of Section 2.2.2 (abbreviated as "P") and the recency statistic of Section 2.2.5 (abbreviated as "R"). Each statistic has a single associated parameter, respectively quantifying the tendency of responders to direct calls toward those whom they have called in the past, and the tendency of responders to direct calls toward those who have called them most recently. In the positive-parameter case, both may be thought of as arising from cognitive mechanisms, with the most cognitively available targets being those toward whom action has been recently directed and those from whom action has been most recently received. Negative parameter values, on the other hand, would suggest other mechanisms at work: the most obvious would be novelty-seeking, which could be reflective of the use of the responder network to search for persons or information within a rapidly changing task environment.

3.2.5. *Conversational Norms*

Finally, we note that radio conversation is a relatively structured form of talk, both for technical and institutional reasons. Strong local reciprocity (i.e., call-response sequences) is to be expected, along with "hand-offs" in which one party transfers contact to another when leaving a conversation. Such conversational norms should produce local dependence of the form captured by the P-shift statistics of Section 2.2.4. Of those listed, we here employ effects for shifts of the form AB-BA (reciprocation), AB-AY and AB-XB (persistence of source or target), AB-BY ("handing off" of communication), and AB-XA (source "attraction"). All are denoted by their Gibson initials ("ABBA," "ABXB," etc.), with the full set of five parameters noted by the initials "PS." The remaining possible shift AB-XY (signifying termination of a conversation) is omitted as a reference category (along with the implicit "nonshift" pattern AB-AB, which is extremely rare in this data set). As all potential targets are also potential senders for the systems considered here, the seven shifts involving nonsending targets are *á priori* excluded.

3.3. *Software Implementation*

Due to the specialized algorithms required to compute the sufficient statistics for relational event models with nontrivial structure, fitting relational event models generally requires special-purpose statistical software. Estimation for all models presented here was conducted via a dedicated library (relevent) written by the author for use with the R statistical computing environment (R Core Development Team 2007); portions of this library also make use of the sna library for network analysis (Butts 2007c). The relevent library is available from the author upon request.

3.4. *Model Selection*

We begin our investigation of the WTC data by fitting a range of models using the effects enumerated above. In each case, parameter estimates were obtained using maximum likelihood under the ordinal time model; the latter was employed due to the fact that exact temporal information was not available for this data set. Size descriptives for the six transcripts treated here are shown in the first two lines of Table 1, where N refers to the number of actors within the network, and M refers to the number of distinct communications recorded. (M is thus the most natural quantity measure for this data.)

Proceeding from the first two rows, all subsequent entries within Table 1 consist of BIC scores (Wasserman 2000) arising from the

TABLE 1
Data Size and BIC Statistics for the Fitted Relational Event Models

Network	PATH Radio	Newark Maint	Newark Police	NJSPEN 2	Newark CPD	WTC Police
N	28	25	24	26	46	35
M	70	77	83	149	271	481
Null	927.93	985.13	1048.05	1930.14	4138.33	6812.60
P	755.99	702.57	786.26	1684.74	3796.23	5754.44
R	659.36	521.08	650.49	1431.95	2946.52	4081.38
T	941.95	999.79	1060.45	1780.55	4034.06	5853.89
PS	512.57	309.80	361.36	1115.52	2001.39	2493.83
PA	902.86	901.04	1021.68	1711.58	3766.50	5703.66
FE	920.27	902.58	1041.14	1381.78	3337.86	4308.54
P+R+T+PS	517.00	331.57	379.95	1040.60	1955.18	2289.74
P+R+T+PS+PA	520.64	333.54	379.57	1041.73	1946.23	2245.71
P+R+T+PS+FE	607.69	419.13	470.36	1008.54	2009.70	2308.08
P+R+T+PS+PA+FE	610.71	423.47	469.99	1011.26	2014.76	2313.65

indicated model/data combination. Models are listed by effects, with codes corresponding to fixed effects (FE), persistence (P), preferential attachment (PA), P-shifts (PS), recency (R), and triads (T) as described above. The null model (listed in the eponymous row) treats all events as equiprobable, and thus serves as a reference for the other models. The next block of models (represented by single terms) includes only one effect in each case and can thus be interpreted as providing evidence of marginal effects. Finally, the third block seeks to combine effects in a manner that facilitates the analysis of hub formation. For this purpose, persistence, recency, P-shifts, and the triadic effects are taken as "controls," with the two major alternatives being preferential attachment (PA) and fixed effects (FE). By investigating BIC scores across models, we can thus evaluate the extent to which one mechanism versus another appears to be providing a more parsimonious account of the data.

Looking across the BIC values of Table 1, a number of patterns clearly emerge. The first, and most striking, is the strong impact of local rules (as implemented via P-shifts) on structural dynamics: no preferred model omits the PS terms, and in three networks these effects alone generate the best-fitting model. The opposite pattern is exhibited by the triad statistics, which had very little success in explaining most of the data sets. The cumulative (P and PA) terms clearly do have some impact on network dynamics, but that impact is fairly weak compared with recency and P-shift effects; in neither case was either able to unseat the marginal R or PS models as the more favored option. As marginals, the fixed effect terms prove generally more powerful than the cumulative terms for the longer transcripts, though not for the shorter ones. Once the control terms are added, however, the BIC favors the addition of preferential attachment over fixed effects in all six networks (though the controls+PA model is itself approximately equal or inferior to the controls only model for four of six cases). This implies that much of the apparent impact of unobserved heterogeneity is in fact the result of cognitive effects and/or local rules, as suggested by Gibson (2003, 2005). Preferential attachment effects may play at least some role in the communication dynamics, though they enter into the BIC-optimal model in only two of the six channels. In interpreting these results, it should be borne in mind that the BIC is well-known to be conservative as model selection index, exhibiting a strong tendency to favor smaller models (Wasserman 2000). As the FE terms add a very large number

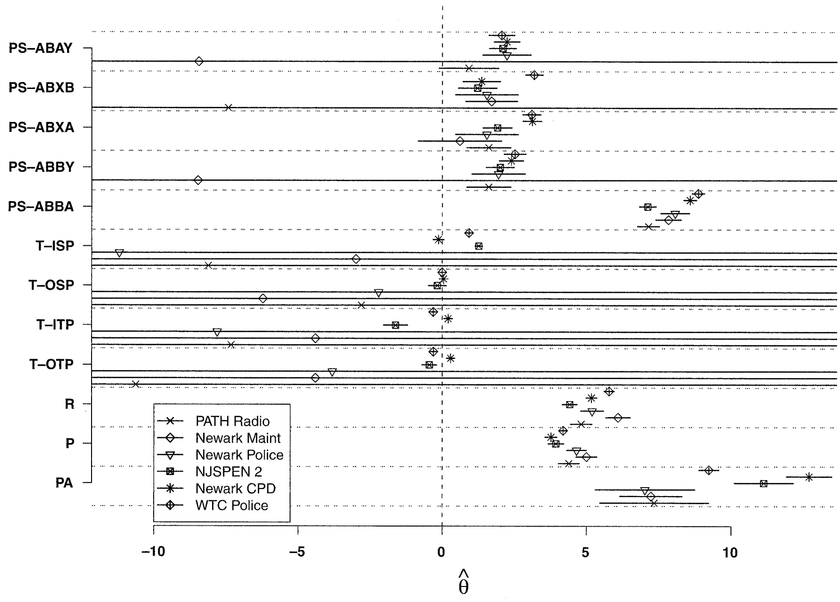

FIGURE 1. Parameter estimates and approximate 95% confidence intervals, marginal models.

of parameters to the model, they are heavily penalized by the BIC, requiring a much larger improvement in goodness-of-fit to achieve the same score as a single-parameter effect such as PA. Nevertheless, the fact that the FE terms are outshone by the PA effect (and/or the controls alone) suggests that latent heterogeneity is less critical to communication dynamics than endogenous effects, at least for the channels considered here.[3]

3.5. *Parameter Estimates*

To get a better idea of the dynamics of communication at the WTC, we now turn to the estimated parameters for the relational event model. Figure 1 shows MLEs (and associated asymptotic 95 percent

[3]AIC-optimal models include FE terms only for the three largest networks; PA terms are included for NJSPEN 2. For the three smaller networks, the AIC and BIC agree except for PATH Radio, where the AIC-optimal model adds the R+P+T controls.

confidence intervals) for the marginal P-shift (PS-ABAY, PS-ABXB, PS-ABXA, PS-ABBY, and PS-ABBA), triadic (T-ISP, T-OSP, T-ITP, T-OTP), recency (R), persistence (P), and preferential attachment (PA) effects; although included in the same figure for comparison, each effect category was fit independently to each transcript.[4] Consistent with Table 1, we observe reasonably strong and systematic marginal effects for the P-shifts (especially AB-BA), recency, persistence, and preferential attachment. All are positive where significant, suggesting marginal tendencies toward reciprocity, persistence in selection of communication targets, and preferential targeting of actors with higher levels of prior communication activity. Triadic effects, however, are more varied: significant effects are observed for only three out of the six transcripts, and little consistency is observed in strength or direction of effect. That said, the three shorter transcripts provide little information regarding triadic structure (as reflected in the large standard errors), leaving open the possibility of subtle effects beneath the detection threshold of the model. While we thus cannot rule out the possibility of consistent triadic biases in the WTC communication dynamics, the data do not provide evidence in support of this assertion.

While the marginal effects shown in Figure 1 appear intuitive, they may also be misleading: Real social systems involve multiple, interacting mechanisms, whose joint effects can be nonobvious. As Table 1 indicates, the BIC-preferred models for all but the shortest transcripts incorporate multiple effects, which must be considered jointly in order to obtain realistic estimates. At the opposite extreme from Figure 1, Figure 2 once again shows parameter estimates for P-shift, triadic, recency, persistence, and preferential attachment effects, this time from joint models in which all effects are included (fixed effects are entered but not shown). These estimates indeed paint a very different picture: Once heterogeneity in activity level is controlled for, the "cumulative" mechanism of preferential attachment either loses significance or reverses direction. As this suggests, attempts to infer preferential attachment effects without considering other factors may misconstrue its impact—in particular, individual-level heterogeneity and preferential attachment clearly substitute for one another to some extent.

[4]95% confidence intervals are based on an asymptotic z approximation, with standard errors derived from the inverse information matrix at the MLE. Some confidence intervals have been truncated for clarity of display.

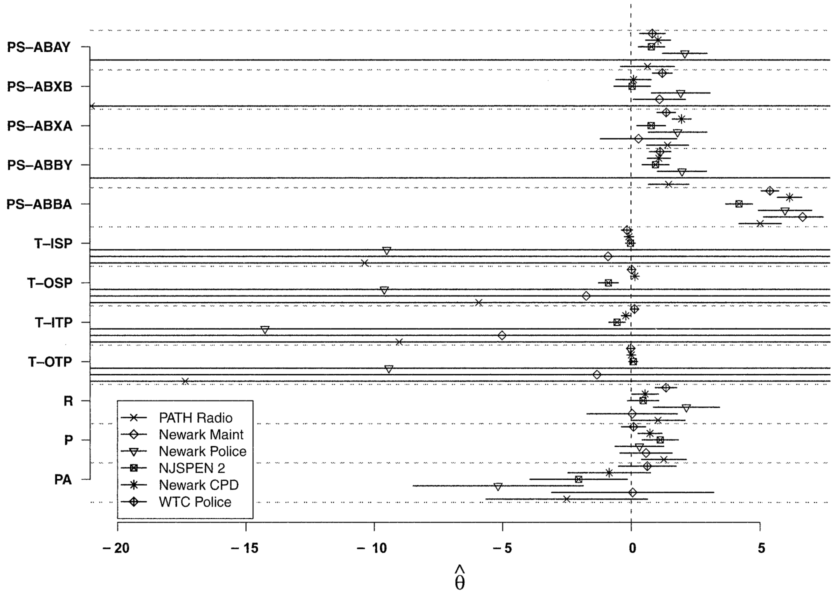

FIGURE 2. Parameter estimates and approximate 95% confidence intervals, joint models.

Having made the point that marginals are not uniformly to be trusted here, we now turn to a more detailed examination of the models most favored by the BIC. Parameter estimates, approximate standard errors, and associated statistics for the six models are shown in Table 2. (Estimates for NJSPEN 2 also include FE terms, which are not shown here.) As noted above, the only effects that enter into the models for the three smallest transcripts are the P-shifts, which are generally dominated by a strong reciprocity effect. As seen in Figure 1, substantial uncertainty exists regarding shift parameters for extremely rare transitions; although the exact values of these parameters are difficult to estimate from the limited data available, they are unlikely to be large given the infrequency of the events in question. (Occasional instances of shift parameters that are identical to within rounding error are similarly due to the discreteness of the data.) Despite the limited data, we can see a strong and consistent reciprocity effect across the three channels: reciprocating communications in these transcripts have over 1600 times the hazard of communications that start new conversations, or which do not induce a P-shift. For the three longer transcripts, we continue to see a strong AB-BA effect, bolstered in two out of three cases by a

TABLE 2
Parameter Estimates for BIC-Selected Models, by Channel

	PATH Radio					Newark Maint					Newark Police				
	$\hat{\theta}$	s.e.	z	Pr(>|z|)		$\hat{\theta}$	s.e.	z	Pr(>|z|)		$\hat{\theta}$	s.e.	z	Pr(>|z|)	
PS-ABAY	0.918	0.524	1.751	0.08		−8.447	65.448	−0.129	0.8973		2.236	0.425	5.262	0^a	***
PS-ABXB	−7.425	32.401	−0.229	0.8188		1.707	0.456	3.739	2e-04	***	1.543	0.553	2.791	0.0053	**
PS-ABXA	1.611	0.387	4.161	0^a	***	0.607	0.736	0.824	0.4098		1.543	0.553	2.791	0.0053	**
PS-ABBY	1.611	0.387	4.16	0^a	***	−8.473	66.324	−0.128	0.8984		1.948	0.471	4.132	0^a	***
PS-ABBA	7.146	0.194	36.746	0^a	***	7.838	0.224	35.054	0^a	***	8.068	0.252	31.985	0^a	***
	Null Dev: 927.926, Residual Dev: 491.326					Null Dev: 985.127, Residual Dev: 288.083					Null Dev: 1048.049, Residual Dev: 339.269				

	NJSPEN 2[b]					Newark CPD					WTC Police				
	$\hat{\theta}$	s.e.	z	Pr(>|z|)		$\hat{\theta}$	s.e.	z	Pr(>|z|)		$\hat{\theta}$	s.e.	z	Pr(>|z|)	
PS-ABAY	0.869	0.258	3.37	8e-04	***	2.07	0.23	8.985	0^a	***	1.904	0.231	8.242	0^a	***
PS-ABXB	−0.088	0.355	−0.248	0.8042		0.428	0.354	1.209	0.2265		1.164	0.2	5.817	0^a	***
PS-ABXA	0.659	0.275	2.396	0.0166	*	2.394	0.179	13.357	0^a	***	1.69	0.18	9.383	0^a	***
PS-ABBY	1.009	0.263	3.831	1e-04	***	2.175	0.22	9.87	0^a	***	2.276	0.196	11.599	0^a	***
PS-ABBA	4.145	0.261	15.877	0^a	***	6.431	0.225	28.624	0^a	***	5.837	0.168	34.805	0^a	***
T-ISP	−0.032	0.095	−0.335	0.7377		0.094	0.087	1.084	0.2785		0.187	0.091	2.047	0.0407	*
T-OSP	−0.843	0.19	−4.449	0^a	***	0.182	0.026	7.038	0^a	***	0.049	0.012	3.976	1e-04	***
T-ITP	−0.558	0.159	−3.517	4e-04	***	−0.191	0.084	−2.283	0.0224	*	0.082	0.054	1.526	0.1271	
T-OTP	0.048	0.085	0.568	0.57		0.021	0.066	0.316	0.7517		−0.226	0.063	−3.598	3e-04	***
R	0.459	0.306	1.501	0.1335		1.792	0.234	7.666	0^a	***	2.552	0.184	13.877	0^a	***
P	0.815	0.325	2.508	0.0121	*	0.067	0.222	0.301	0.7632		−0.86	0.214	−4.027	1e-04	***
PA						4.492	0.827	5.431	0^a	***	5.073	0.487	10.414	0^a	***
	Null Dev: 1930.138, Residual Dev: 828.401					Null Dev: 4138.335, Residual Dev: 1879.000					Null Dev: 6812.604, Residual Dev: 2171.598				

$^*p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$, $^a p <$ 1e-4, [b]25 FE terms omitted.

significant recency effect. Persistence and triad effects are not strongly consistent, although we do see significant impacts of these parameters within particular groups. Of more interest is the PA effect. Although present in only two channels, it is noteworthy that the effects on those channels are large and positive. Since these models also control for the other basic effects, our earlier intuition that the substitution effect observed is between the PA and FE terms would appear to be confirmed. More fundamentally, these results would seem to suggest that where preferential attachment exists, it is positive in effect—but its impact is substantially less fundamental than low-level factors such as turn-taking and recency.

Clearly, the above analyses suggest that endogenous mechanisms play a stronger role than responder-level heterogeneity in activity levels (as captured by the fixed effect parameters) in determining communication network structure. Despite this, the FE terms do show significant variability in the joint models (not shown), and it bears investigating whether some covariate might account for such heterogeneity as is present. While this heterogeneity could stem from many sources—including differences in context, training, or cognitive/emotional state—we will here consider only the possible influence of institutionalized coordinative roles. Intuitively, we may expect responders with such roles to act as hubs within the communication network, thus displaying higher levels of activity (net of other processes) than actors without such roles. On the other hand, demands for emergent coordination (Dynes 2003) may render such roles largely irrelevant during the immediate aftermath of a high-consequence event. To assess this possibility, we compare fixed effect estimates under the joint model for responders holding institutionalized coordinative roles with those for other responders. Figure 3 shows boxplots for activity level effects by institutional status, for all six WTC networks. As the figure suggests, the impact of institutionalized coordinative roles is weak at best: Although medians for the institutionalized coordinators are higher in five out of the six networks, the mean differences are not significant in any case. Pooled mean differences are similarly insignificant ($z = -1.26$, $p = 0.21$). Our dynamic analysis thus reinforces the findings of Petrescu-Prahova and Butts (2005), whose static analysis of WTC communications found evidence that centrality within responder radio communication networks was largely due to factors other than institutional status.

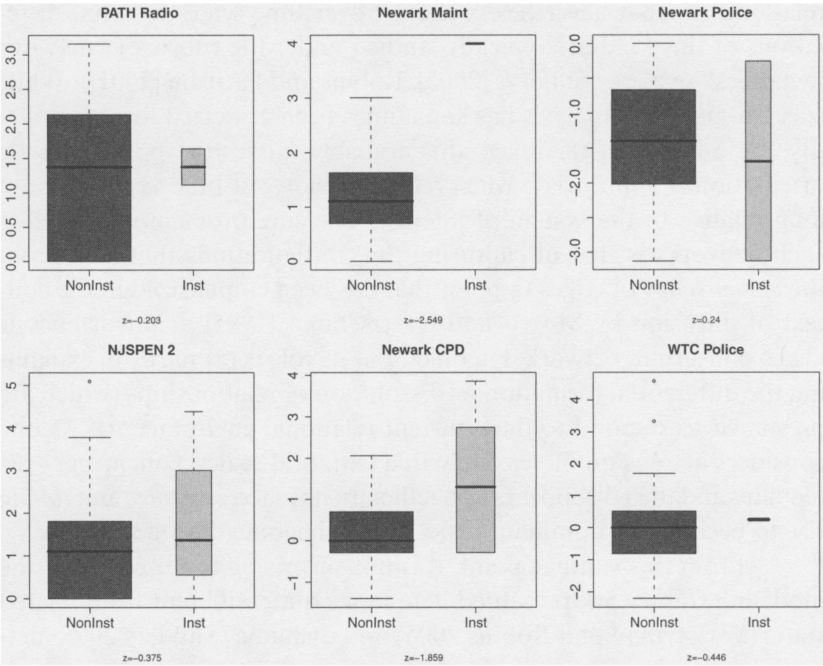

FIGURE 3. Fixed effect parameter distribution by institutional status, joint models.

4. CONCLUSION

In this paper, we have introduced a stochastic model for intertemporal behavioral data, based on a relational event formalism. Using the simplifying assumption of piecewise constant hazards, we are able to construct a fairly broad modeling framework that can be applied to data with either exact or ordinal timing information. A wide range of mechanisms can be evaluated within this framework, of which several are illustrated here; once specified, parameters associated with the direction and strength of these effects can be readily estimated using maximum likelihood methods.

As we have stressed, relational events are temporally local phenomena, and thus represent the opposite end of the temporal continuum from the (relatively) long-term structures that have formed the primary subject matter of classical network analysis (e.g., see Wasserman and Faust [1994]). Between these extremes lie temporally extensive relationships that nevertheless change over time scales of interest; relations of this kind are typically studied under the rubric of "network dynamics" (e.g., see Snijders [2005]; Robins and Pattison [2001]). While classical network analysis has sometimes been criticized for its implicitly static frame of reference, this arguably misses the point: a static orientation *is* appropriate when relations evolve at time scales that are long relative to the system of interest. The core problem in modeling such networks is that of capturing the configurations that arise from the *concurrency* of edges (a point that has been emphasized in the context of diffusion by Morris and Kretzschmar [1995]). Concurrency is also a concern in network dynamics, but its role is primarily in explaining the differential formation or dissolution of relationships (which are presumed to respond to their current relational environment). As one considers actions on increasingly fine temporal scales, concurrency attenuates and then disappears altogether. In its place, *sequence* and *timing* rise to become the dominant concepts of phenomenal concern.

From this vantage point, it is not surprising to observe that the modeling framework presented here looks quite different from related static (Wasserman and Robins 2005) and dynamic (Snijders 2005) network approaches. Similarly, the challenges that are encountered in modeling relational events are quite different from those encountered when modeling temporally extensive relationships. The ability to impose sequential dependence on all events, in particular, avoids the more complex, simultaneous structures of dependence that emerge from realistic models with concurrent relationships. In this respect, a loose analogy may be made between the relation event/network modeling distinction and the distinction between modeling temporal and spatial autocorrelation. Temporal autocorrelation, while nontrivial, has proved a much easier target than its spatial counterpart. The reason for this is largely the same: Strict temporal ordering greatly reduces the range of plausible models and tends to lead to models with simpler structure. Just as we would not replace a spatial autocorrelation model with a temporal autocorrelation model because the latter is simpler, convenience is not a valid rationale for attempting to shoehorn a high-concurrency network process into a relational event framework. That said, we should certainly make use of the latter's (relative) simplicity where appropriate, and it may be conjectured that scientific progress may be more rapid for phenomena that admit relational event structure than those that are temporally extensive.

To summarize our substantive findings, our analysis of six transcripts from the World Trade Center disaster suggests that a combination of cognitive/behavioral effects and local rules—not latent heterogeneity or preferential attachment—is the key driver of the dynamic behavior of WTC radio communication networks. As expected, strong reciprocity effects were observed for all six transcripts; triadic effects, on the other hand, did not appear to play a large role in driving communication dynamics. Although we cannot currently identify the source of that responder heterogeneity that is observed here, our analysis indicates that institutionalized coordinative roles have little explanatory power in this regard. This is consistent with the hypothesis that emergency phase responder activity at the WTC was largely driven by idiosyncratic, situational factors that overwhelmed prior organization. On the other hand, the persistence of simple microdynamic rules such as conversational norms speaks strongly to the robustness of low-level behavior to substantial disruption. These observations are broadly consistent with the findings of the 9/11 Commission (National Commission on Terrorist Attacks Upon the United States 2004) and with the conclusions reached by Butts et al. (2007) and Petrescu-Prahova and Butts (2005) based on aggregate analyses, though it should be cautioned that alternative explanations may also exist. Additional analyses with a larger body of data should shed further light on this issue.

In closing, we note that the "computational revolution" that swept through the social sciences during the 1980s and 1990s seems now to be enabling a new "statistical turn" in the modeling of complex social systems. Research in this vein does not retreat from the systematic treatment of dependence and interaction that has served as the hallmark of computational modeling over the past several decades, but it is likewise attentive to the need for such models to have a principled inferential foundation. By leveraging innovations in areas such as statistical simulation and exponential family theory, it is increasingly possible for researchers to achieve both goals—a result that would surely have pleased early formal theorists such as Coleman (1964), who repeatedly stressed the importance of bringing together formal models and empirical data. The model family presented in this paper is one attempt at such a union of theory and method, and it is hoped that this work will encourage others to undertake similar efforts.

APPENDIX: GLOSSARY OF SYMBOLS

The following list contains all prominent functions or other quantities used within this paper, in order of introduction within the text; it may serve as a useful reference when reviewing formal developments within the paper.

$\mathcal{S}$	Sender set
$\mathcal{R}$	Receiver set
$\mathcal{C}$	Action type set
$a = (i, j, k, t)$	Action, or relational event
$s(a)$	Sender of action a
$r(a)$	Receiver of action a
$c(a)$	Type of action a
$\tau(a)$	Time of action a
a_0	Null action
$A_t = \{a_i : \tau(a_i) \leq t\}$	History of all events occurring by time t
$\mathbb{A}(A_t)$	Support set for events at time t, given the current event history
$f(x)$	PDF of random variable X at x
$F(x)$	CDF of random variable X at x
$S(x)$	Survival function for random variable X at x
$h(x)$	Hazard function for random variable X at x
$p(x)$	Likelihood of random variable X at x; assumed to be a PMF where X is discrete, or a PDF where X is continuous
M	Number of non-null events in A_t
X_a	Covariate set associated with event a
θ	Vector of real-valued parameters
$\lambda_{aA_t\theta} = \lambda(s(a), r(a), c(a), X_a, A_t, \theta)$	Hazard of event a given the event's properties, the current event history, model parameters, and covariates
λ_0	Baseline hazard (or global pacing constant)
$u(s(a), r(a), c(a), X_a, A_t)$	The vector of sufficient statistics for the hazard of a at time t

$I(x)$	The dichotomous indicator function for event x
u_{FE_m}	Fixed effect statistic for actor m
$d(i, j, A_k)$	The number of events sent from i to j by time k
$d(i, A_k)$	The number of events sent from i by time k
u_{P}	Persistence statistic
u_{PA}	Preferential attachment statistic
SS	For a sequential action pair (a', a), the assertion $s(a') = s(a)$
RR	For a sequential action pair (a', a), the assertion $r(a') = r(a)$
SR	For a sequential action pair (a', a), the assertion $s(a') = r(a)$
RS	For a sequential action pair (a', a), the assertion $r(a') = s(a)$
T, T'	For a sequential action pair (a', a) and restricted receiver set $\mathcal{R}' = \mathcal{R} \cap \mathcal{S}$, the respective assertions $r(a) \in \mathcal{R}'$, $r(a) \in \mathcal{R}'$
$u_{\mathrm{AB-BA}}$	P-shift statistic of type AB-BA
$u_{\mathrm{AB-B0}}$	P-shift statistic of type AB-B0
$u_{\mathrm{AB-BY}}$	P-shift statistic of type AB-BY
$u_{\mathrm{A0-X0}}$	P-shift statistic of type A0-X0
$u_{\mathrm{A0-XA}}$	P-shift statistic of type A0-XA
$u_{\mathrm{A0-XY}}$	P-shift statistic of type A0-XY
$u_{\mathrm{AB-X0}}$	P-shift statistic of type AB-X0
$u_{\mathrm{AB-XA}}$	P-shift statistic of type AB-XA
$u_{\mathrm{AB-XB}}$	P-shift statistic of type AB-XB
$u_{\mathrm{AB-XY}}$	P-shift statistic of type AB-XY
$u_{\mathrm{A0-AY}}$	P-shift statistic of type A0-AY
$u_{\mathrm{AB-A0}}$	P-shift statistic of type AB-A0
$u_{\mathrm{AB-AY}}$	P-shift statistic of type AB-AY
u_{R}	Recency statistic
u_{OTP}	Outgoing two-path statistic
u_{ITP}	Incoming two-path statistic
u_{OSP}	Outgoing shared partner statistic
u_{ISP}	Incoming shared partner statistic
$\hat{\theta}$	The MLE of θ

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